PARMA CITY SCHOOL DISTRICT
COURSE OF STUDY
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MATHEMATICS 6-12

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INTRODUCTION/BACKGROUND/PURPOSE/RATIONALE

In 2010, the State Board of Education adopted Ohio's Learning Standards in Mathematics as a guide to teaching and learning in the classroom. The kindergarten-grade 12 standards have been fully in use in Ohio classrooms since the start of the 2014-2015 school year. In early 2016, educators statewide began assisting the Ohio Department of Education in updating Ohio's Learning Standards in Mathematics. To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education. The board adopted the proposed revisions for Ohio's Learning Standards for English Language Arts in early winter 2017.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

TWO SETS OF STANDARDS FOR MATHEMATICS

Ohio's Learning Standards for Mathematics are comprised of two sets of complementary standards: Standards for Mathematical Practice and Standards for Mathematical Content. BOTH sets of standards should be taught to and assessed for mastery in K-12 classrooms through a variety of evidence-based methods.

STANDARDS FOR MATHEMATICAL PRACTICE K-12

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation $(y - 2)/(x - 1) = 3$.

Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x2 + x + 1)$, and $(x - 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT K-12

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

INTENDED LEARNING OUTCOMES AND LEARNING PROGRESSIONS

The concept of learning progressions was critical in the development, review, and revision of Ohio’s Learning Standards (OLS). Ohio’s learning progressions were developed during Ohio’s international benchmarking project and provided guidance to the writing and revision committees of Ohio’s Learning Standards. Ohio believes that the concept of learning progressions is important for the understanding and coherence of mathematical topics within and across the grade levels. The Ohio Department of Education has reformatted Ohio’s Learning Standards by domains to show the progression of concepts and skills across the grade levels. The document entitled *Ohio’s K-8 Learning Progressions* February 2017 provides staff with clear indications of the progression of conceptual standards categories as well as the specific standards for each grade level that fall under each category. Staff should use this document when writing courses of study, curriculum maps and unit plans.

Ohio's Learning Standards for Mathematics include critical areas for instruction in the introduction to each grade, kindergarten through grade 8 as well as Algebra 1 and Geometry for the high school level. The critical areas are designed to bring focus to the standards at each grade by describing the big ideas that educators can use to build their curriculum and to guide instruction. The grade-level introductions include at least two and no more than five critical areas for each grade. These critical areas identify the intended overall learning outcomes for grades K-8 and are cited within each grade level section standard detail use parenthetical notation (e.g. CA1, CA2, etc.).

According to the IES practice guide entitled *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, the understanding of fractions is essential for understanding algebraic concepts and other higher levels of math (p. 6). Furthermore, knowledge of fraction concepts presents a larger gap for students in the United States than whole number concepts when compared to other countries such as East Asia (p. 6). The panel of this practice guide suggest these deficiencies are the result of the following conceptual misconceptions:

- Not viewing fractions as numbers at all, but rather as meaningless symbols that need to be manipulated in arbitrary ways to produce answers that satisfy a teacher.
- Focusing on numerators and denominators as separate numbers rather than thinking of the fraction as a single number. Errors such as believing that 3/8 > 3/5 arise from comparing the two denominators and ignoring the essential relation between each fraction's numerator and its denominator.
- Confusing properties of fractions with those of whole numbers. This is evident in many high school students' claim that just as there is no whole number between 5 and 6, there is no number of any type between 5/7 and 6/7.

The panel recommends that teachers employ the following strategies and supports in student fractions concept development:

As student progress through the grade level standards in mathematics and apply their knowledge to different problem solving tasks, it is important for educators to work with student to improve problem solving skills. In the IES practice guide entitled *Improving Mathematical Problem Solving in Grade 4 through 8*, the panel recommends that teachers employ the following strategies and supports in student problem solving skill development:

Recommendation 1: Prepare problems and use them in whole-class instruction.

• Include both routine and non-routine problems in problem-solving activities (will vary based upon each student's level of experience with problem-solving).
    ○ *Routine:* can be solved using methods familiar to students by replicating previously learned methods in step-by-step fashion.
    ○ *Non-routine:* problems for which there are no predictable or well-rehearsed approaches nor an explicit suggestion of the pathway in the task, task instructions or a worked-out example.
• Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language.
• Consider students' knowledge of mathematical content when planning lessons.

Recommendation 2: Assist students with in monitoring and reflecting on the problem-solving process.
• Provide students with a list of prompts to help them monitor and reflect during the problem-solving process. • Model how to monitor and reflect on the problem-solving process. • Use student thinking about a problem to develop students' ability to monitor and reflect.
Recommendation 3: Teach students how to use visual representations.
• Select visual representations that are appropriate for students and the problems they are solving. • Use think-alouds and discussions to teach students how to represent problems visually. • Show students how to convert the visually represented information into mathematical notation.
Recommendation 4: Expose students to multiple problem-solving strategies.
• Provide instruction in multiple strategies. • Provide opportunities for students to compare multiple strategies in worked examples. • Ask students to generate and share multiple strategies for solving a problem.
Recommendation 5: Help students recognize and articulate mathematical concepts and notation.
• Describe mathematical concepts and notation, and related them to the problem-solving activity. • Ask students to explain each step used to solve a problem in a worked example. • Help students make sense of algebraic notation.

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These recommendations and practices will benefit students K-12. As rigor in coursework increases, it is important for educators to review and continue to implement these foundational approaches to mathematics instruction. They provide a framework for the development of an effective approach to problem solving from the basic to the most abstract.

GAISE

The Ohio Content Standards for Mathematics emphasize the importance of incorporating the recommendations in the Guide for Assessment and Instruction in Statistics Education (GAISE) report when teaching PK-12 mathematical concepts related to statistics. The GAISE report recommends developmental levels of the learning process as applied to statistics education as follows regardless of grade level. These levels are applied to students not by age, but rather their level of experience with statistics.

Table 1: The Framework

Process Component	Level A	Level B	Level C
I. Formulate Question	***Beginning awareness of the statistics question distinction*** Teachers pose questions of interest Questions restricted to the classroom	***Increased awareness of the statistics question distinction*** Students begin to pose their own questions of interest Questions not restricted to the classroom	***Students can make the statistics question distinction*** Students pose their own questions of interest Questions seek generalization
II. Collect Data	Do not yet *design for differences* Census of classroom Simple experiment	Beginning awareness of *design for differences* Sample surveys; begin to use random selection Comparative experiment; begin to use random allocation	Students make *design for differences* Sampling designs with random selection Experimental designs with randomization
III. Analyze Data	*Use* particular properties of *distributions* in the context of a specific example Display variability within a group Compare individual to individual Compare individual to group Beginning awareness of group to group Observe association between two variables	Learn to *use* particular properties of *distributions* as tools of analysis Quantify variability within a group Compare group to group in displays Acknowledge sampling error Some quantification of association; simple models for association	Understand and *use distributions* in analysis as a global concept Measure variability within a group; measure variability between groups Compare group to group using displays and measures of variability Describe and quantify sampling error Quantification of association; fitting of models for association

Process Component	Level A	Level B	Level C
IV. Interpret Results	Students do not look *beyond the data* No generalization beyond the classroom Note difference between two individuals with different conditions Observe association in displays	Students acknowledge that *looking beyond the data* is feasible Acknowledge that a sample may or may not be representative of the larger population Note the difference between two groups with different conditions Aware of distinction between observational study and experiment Note differences in strength of association Basic interepretation of models for association Aware of the distinction between association and cause and effect	Students are able to *look beyond the data* in some contexts Generalize from sample to population Aware of the effect of randomization on the results of experiments Understand the difference between observational studies and experiments Interpret measures of strength of association Interpret models of association Distinguish between conclusions from association studies and experiments
Nature of Variability	Measurement variability Natural variability Induced variability	Sampling variability	Chance variability
Focus on Variability	Variability within a group	Variability within a group and variability between groups Covariability	Variability in model fitting

It is expected that teachers of mathematics apply the recommendations contained in the GAISE report when planning and delivering lessons as well as assessing student mastery of statistics education concepts.

LEARNING TARGETS & ACADEMIC VOCABULARY

The following section outlines the specific state and local content standards each teacher should teach and assess for mastery as outlined by the required sequencing depicted in curriculum maps provided each year by the Department of Curriculum and Instruction. In addition, each teacher should teach and assess for mastery the target academic vocabulary words contained within the standards for each grade level/course as directed and provided by the Department of Curriculum and Instruction. The Department of Curriculum & Instruction will request feedback periodically from staff in regard to any suggested revisions to sequencing and/or local standards language or target academic vocabulary.

The standards offer a focus for instruction each year and help ensure that student gain adequate experience with a range of mathematical concepts and practice skills through a variety of appropriately aligned math tasks. *Students advancing through the grades should meet each year's specific standards and retain or further develop skills and understandings mastered in preceding grades.*

These standards encourage fostering students' understanding and working knowledge of mathematical concepts and practices. Educators should differentiate instruction; **the point is to teach students what they need to learn and not what they already know--- to discern when particular children or activities warrant more or less attention.**

It is expected that staff keep abreast of current evidence-based practices that have a strong and/or moderate evidence to support effectiveness of the strategy and/or resource. The teaching and reinforcement of academic vocabulary is paramount to student academic success and a promising practice for closing learning gaps with at risk subgroups of learners (Marzano, 2001; Marzano, 2005; Marzano, 2010; Marzano & Simms, 2013; Institute of Education Sciences, 2016).

GRADE 6

In Grade 6, instructional time should focus on five critical areas:

Critical Area 1(CA1): Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

Critical Area 2(CA2): Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

Critical Area 3(CA3): Writing, interpreting, and using expressions and equations Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

Critical Area 4(CA4): Developing understanding of statistical problem solving Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. The GAISE model is used as a statistical problem solving framework. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (range and interquartile range) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, gaps, peaks, and outliers in a distribution, considering the context in which the data were collected.

Critical Area 5(CA5): Solving problems involving area, surface area, and volume Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

GRADE 6

RATIO AND PROPORTIONAL RELATIONSHIPS ● ***Understand ratio concepts and use ratio reasoning to solve problems.***
Standard
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." **(CA1)**
6.RP.2 Understand the concept of a unit rate a/b associated with ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger." **(CA1)**
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams**[G]** or equations. **(CA1)** a. Make tables of equivalent ratios relating quantities with whole- number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
THE NUMBER SYSTEM ● ***Apply and extend previous understandings of multiplication and division to divide fractions by fractions.***
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models**[G]** and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 pound of chocolate equally? How many 3/4 cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi? **(CA2)**

[G:] Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

• ***Compute fluently with multi-digit numbers and find common factors and multiples.***
Standard
6.NS.2 Fluently[G] divide multi-digit numbers using a standard algorithm[G] **(CA2)**
6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation. **(CA2)**
6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2). **(CA2)**
• ***Apply and extend previous understandings of numbers to the system of rational numbers.***
6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **(CA2)**
6.NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **(CA2)** a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., − (−3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
6.NS.7 Understand ordering and absolute value of rational numbers. **(CA2)** a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right.* b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write −3°C > −7°C to express the fact that −3°C is warmer than −7 °C.* c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars.* d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars.*
6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. **(CA2)**

G: Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

EXPRESSIONS AND EQUATIONS ● ***Apply and extend previous understandings of arithmetic to algebraic expressions.***
Standard
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.**(CA3)**
6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. **(CA3)** a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 − y.* b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*
● ***Apply and extend previous understandings of arithmetic to algebraic expressions, continued.***
6.EE.3 Apply the properties of operations to generate equivalent expressions*. For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **(CA3)**
6.EE.4 Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* **(CA3)**
● ***Reason about and solve one-variable equations and inequalities.***
6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **(CA3)**
6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **(CA3)**
6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q, and x are all nonnegative rational numbers. **(CA3)**
6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **(CA3)**
● ***Represent and analyze quantitative relationships between dependent and independent variables.***
6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.* **(CA3)**

GEOMETRY ● ***Solve real-world and mathematical problems involving area, surface area, and volume.***
Standard
6.G.1 Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems. **(CA5)**
6.G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell \cdot w \cdot h$ and $V = B \cdot h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **(CA5)**
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **(CA5)**
6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **(CA5)**
STATISTICS AND PROBABILITY ● ***Develop understanding of statistical problem solving.***
6.SP.1 Develop statistical reasoning by using the GAISE model: **(CA4)** a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages.* (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)
6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **(CA4)**
6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **(CA4)**
● ***Summarize and describe distributions.***
6.SP.4 Display numerical data in plots on a number line, including dot plots (line plots)**[G], ** histograms, and box plot**[G]** (GAISE Model, step 3) **(CA4)**
6.SP.5 Summarize numerical data sets in relation to their context. **(CA4)** a. Report the number of observations. b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile range**[G]**) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.

GRADE 7

In Grade 7, instructional time should focus on five critical areas:
Critical Area 1(CA1): Developing understanding of and applying proportional relationships Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.
Critical Area 2(CA2): Developing understanding of operations with rational numbers and working with expressions and linear equations Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts, e.g., amounts owed or temperatures below zero, students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.
Critical Area 3(CA3): Solving problems involving scale drawings and informal geometric constructions, angles, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Critical Area 4(CA4): Drawing inferences about populations based on samples Students build on their previous work with statistical problem solving through the use of the GAISE model framework. They summarize and describe distributions representing one population and informally compare two populations. Students interpret numerical data sets using mean absolute deviation. They begin informal work with sampling to generate data sets: learn about the importance of representative samples for drawing inferences and the impact of bias.
Critical Area 5(CA5): Investigating chance Students build upon previous understandings as they develop concepts of probability. They investigate relevant chance events and develop models to determine and compare probabilities. They analyze the frequencies of the experimental results against their predictions, justifying any discrepancies. Students extend their investigations with compound events representing the possible outcomes in tree diagrams, tables, lists, and ultimately through designing and using simulations.

GRADE 7

RATIO AND PROPORTIONAL RELATIONSHIPS ● ***Analyze proportional relationships and use them to solve real-world and mathematical problems.***
Standard
7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks ½ mile in each 1/4 hour, compute the unit rate as the complex fraction*[G] *(1/2)/(1/4) miles per hour, equivalently 2 miles per hour.* **(CA1)**
● ***Analyze proportional relationships and use them to solve real-world and mathematical problems, continued.***
7.RP.2 Recognize and represent proportional relationships between quantities. **(CA1)(CA2)** a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **(CA1)**
THE NUMBER SYSTEM ● ***Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.***
7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. **(CA2)** a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* b. Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.

G: Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

• ***Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers, continued.***
Standard
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **(CA2)** a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions. **(CA2)**
EXPRESSIONS AND EQUATIONS • ***Use properties of operations to generate equivalent expressions.***
7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **(CA2)**
7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. *For example, a discount of 15% (represented by p − 0.15p) is equivalent to (1 − 0.15) p, which is equivalent to 0.85p or finding 85% of the original price.* **(CA2)**
• ***Solve real-life and mathematical problems using numerical and algebraic expressions and equations.***
7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example, if a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar* 9 ¾ *inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **(CA2)**

- ***Solve real-life and mathematical problems using numerical and algebraic expressions and equations, continued.***

Standard

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. **(CA2)(CA3)**

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

GEOMETRY

- ***Draw, construct, and describe geometrical figures and describe the relationships between them.***

7.G.1 Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. **(CA1)(CA3)**

a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale.

b. Represent proportional relationships within and between similar figures.

7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. **(CA3)**

a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

b. Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.

7.G.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **(CA3)**

- ***Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.***

7.G.4 Work with circles. **(CA3)**

a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle.

b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.

7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **(CA3)**

7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **(CA3)**

STATISTICS AND PROBABILITY ● ***Use sampling to draw conclusions about a population.***
Standard
7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population. **(CA4)** a. Differentiate between a sample and a population. b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.
● ***Broaden understanding of statistical problem solving.***
7.SP.2 Broaden statistical reasoning by using the GAISE model: **(CA4)** a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?"* (GAISE Model, step 1) b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)
● ***Summarize and describe distributions representing one population and draw informal comparisons between two populations.***
7.SP.3 Describe and analyze distributions. **(CA4)** a. Summarize quantitative data sets in relation to their context by using mean absolute deviation[G] (MAD), interpreting mean as a balance point. b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot G (line plot), the separation between the two distributions of heights is noticeable.*
● ***Investigate chance processes and develop, use, and evaluate probability models.***
7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around 1/2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event. **(CA5)**
7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.* **(CA5)**

[G:] Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

● ***Investigate chance processes and develop, use, and evaluate probability models, continued.***
Standard
7.SP.7 Develop a probability model[G] and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. **(CA5)** a. Develop a uniform probability model[G] by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations. **(CA5)** a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space[G] for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

[G:] Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

GRADE 8

In Grade 8, instructional time should focus on four critical areas:

Critical Area 1(CA1): Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables graphically or by simple inspection; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Critical Area 2(CA2): Grasping the concept of a function and using functions to describe quantitative relationships Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

Critical Area 3(CA3): Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Critical Area 4(CA4): Working with irrational numbers, integer exponents, and scientific notation Students explore irrational numbers and their approximations. They extend work with expressions and equations with integer exponents, square and cube roots. Understandings of very large and very small numbers, the place value system, and exponents are combined in representations and computations with scientific notation.

GRADE 8

THE NUMBER SYSTEM

- ***Know that there are numbers that are not rational, and approximate them by rational numbers.***

Standard

8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating. **(CA4)**

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. **(CA4)**

EXPRESSIONS AND EQUATIONS

- ***Work with radicals and integer exponents.***

8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3-5 = 3-3 = 1/3^3 = 1/27$. **(CA4)**

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **(CA4)**

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **(CA4)**

8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology. **(CA4)**

- ***Understand the connections between proportional relationships, lines, and linear equations.***

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **(CA1)**

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **(CA1)(CA3)**

- ***Analyze and solve linear equations and pairs of simultaneous linear equations.***

8.EE.7 Solve linear equations in one variable. **(CA1)**

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

● ***Analyze and solve linear equations and pairs of simultaneous linear equations, continued.***
Standard
8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically. **(CA1)** a. Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (Limit solutions to those that can be addressed by graphing.)
FUNCTIONS ● ***Define, evaluate, and compare functions.***
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8. **(CA2)**
8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. **(CA2)**
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line.* **(CA2)**
● ***Use functions to model relationships between quantities.***
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **(CA2)**
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **(CA2)**

GEOMETRY ● ***Understand congruence and similarity using physical models, transparencies, or geometry software.***
Standard
8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). **(CA3)** a. Lines are taken to lines, and line segments are taken to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
8.G.2 Understand that a two-dimensional figure is congruent[G] to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.) **(CA3)**
● ***Understand congruence and similarity using physical models, transparencies, or geometry software, continued.***
8.G.3 Describe the effect of dilations[G], translations, rotations, and reflections on two-dimensional figures using coordinates. **(CA3)**
8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.) **(CA3)**
8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* **(CA3)**
● ***Understand and apply the Pythagorean Theorem.***
8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse. **(CA3)**
8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **(CA3)**
8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. **(CA3)**

[G]: Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

STATISTICS AND PROBABILITY ● ***Investigate patterns of association in bivariate data***
Standard
8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4) **(CA1)**
8.SP.2 Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4) **(CA1)**
8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr. as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4) **(CA1)**
8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated from rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* **(CA1)**

[G]: Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

Mathematical Content Standards for High School

PROCESS

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students. However, standards with a (+) symbol will not appear on Ohio's State Tests.

The high school standards are listed in conceptual categories:

- Modeling
- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★)

High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include the following:

• Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
• Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
• Designing the layout of the stalls in a school fair so as to raise as much money as possible.
• Analyzing stopping distance for a car.
• Modeling savings account balance, bacterial colony growth, or investment growth.
• Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
• Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
• Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.
In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena, e.g., the behavior of polynomials as well as physical phenomena.

MODELING STANDARDS

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

GRADES 9-12 – NUMBER AND QUANTITY STANDARDS

THE REAL NUMBER SYSTEM ● ***Extend the properties of exponents to rational exponents.***
Standard
N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*
N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
● ***Use properties of rational and irrational numbers.***
N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
QUANTITIES ● ***Reason quantitatively and use units to solve problems.***
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★
N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

THE COMPLEX NUMBER SYSTEM

- ***Perform arithmetic operations with complex numbers.***

Standard
N.CN.1 Know there is a complex number □□ such that □□2 = −1, and every complex number has the form □□ + □□□□with □□ and □□ real.
N.CN.2 Use the relation □□2= −1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
(+) N.CN.3 Find the conjugate of a complex number; use conjugates to find magnitudes and quotients of complex numbers.
• ***Represent complex numbers and their operations on the complex plane.***
(+) N.CN.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
(+) N.CN.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (−1 + □□√3)³ = 8 because (−1 + □□√3) has magnitude 2 and argument 120°.*
(+) N.CN.6 Calculate the distance between numbers in the complex plane as the magnitude of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
• ***Use complex numbers in polynomial identities and equations.***
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
(+) N.CN.8 Extend polynomial identities to the complex numbers. *For example, rewrite x^2 + 4 as (x + 2 □□) (x − 2 □□).*
(+) N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

VECTOR AND MATRIX QUANTITIES

- ***Represent and model with vector quantities.***

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.
(+) N.VM.1 Recognize vector[G] quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes, e.g., □□, | □□ |, ||□□||, □□□.
(+) N.VM.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
(+) N.VM.3 Solve problems involving velocity and other quantities that can be represented by vectors.

[G:] Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

● ***Perform operations on vectors.***
Standard
(+) N.VM.4 Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\boldsymbol{vv} - \boldsymbol{ww}$ as $\boldsymbol{vv} + (-\boldsymbol{ww})$, where $-\boldsymbol{ww}$ is the additive inverse of $\boldsymbol{ww}$, with the same magnitude as $\boldsymbol{ww}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
(+) N.VM.5 Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $cc\,(vvx,\, vvy) = (ccccx,\, ccccy)$. b. Compute the magnitude of a scalar multiple $cc\boldsymbol{vv}$ using $\|\|cc\boldsymbol{vv}\|\| = \|cc\|\,\boldsymbol{vv}$. Compute the direction of $cc\boldsymbol{vv}$ knowing that when $\|\,cc\,\|\,\boldsymbol{vv} \neq 0$, the direction of $cc\boldsymbol{vv}$ is either along $\boldsymbol{vv}$ (for $cc > 0$) or against $\boldsymbol{vv}$ (for $cc < 0$).
● ***Perform operations on matrices, and use matrices in applications.***
(+) N.VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
(+) N.VM.7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
(+) N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.
(+) N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
(+) N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication analogous to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
● ***Perform operations on matrices, and use matrices in applications, continued.***
(+) N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
(+) N.VM.12 Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

High School—Algebra

EXPRESSIONS

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.
A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

EQUATIONS AND INEQUALITIES

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable from a set of numbers; the solutions of an equation in two variables from a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.
Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, *A* = ((��1+��2)2*)* *h*, can be solved for *h* using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

CONNECTIONS WITH FUNCTIONS AND MODELING

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

CRITICAL AREAS OF FOCUS ALGEBRA 1

In Algebra 1, instructional time should focus on five critical areas:

CRITICAL AREA OF FOCUS #1 (CA1) Relationships between Quantities and Reasoning with Equations By the end of eighth grade students have learned to solve linear equations in one variable and have applied graphical methods to analyze and solve systems of linear equations in two variables. Now students build on these earlier experiences by analyzing and explaining the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All this work is grounded on understanding quantities and on the relationships between them. Students apply this learning in real-world and modeling situations. Number and Quantity - Quantities Reason quantitatively and use units to solve problems. N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

CRITICAL AREA OF FOCUS #2 (CA2) Linear and Exponential Relationships In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. Students will learn function notation and develop the concepts of domain and range. Their understanding moves beyond viewing functions as processes that take inputs and yield outputs and to viewing functions as objects in their own right followed by an informal introduction of inverse functions. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate or incomplete. Their work includes functions that can be described or modeled by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. Algebra – Reasoning with Equations and Inequalities Solve systems of equations.

CRITICAL AREA OF FOCUS #3(CA3) Descriptive Statistics In middle school, students developed an understanding of statistical problem solving through the format of the GAISE Model. They were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school, students were creating scatterplots and recognizing linear trends in data. Now, they apply those concepts by using the GAISE model in the context of real-world applications. Students develop formal means of assessing how a model fits data. They use regression techniques to describe approximately linear relationships between quantities. Students use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. In Algebra 2/Mathematics 3, students will look at residuals to analyze the goodness of fit.

CRITICAL AREA OF FOCUS #4(CA4) Expressions and Equations Students apply the properties of operations with real numbers, the relationships between the operations, along with the properties of exponents to operations with polynomials. Also, students focus on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

CRITICAL AREA OF FOCUS #5(CA5) Quadratic Functions and Modeling In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to gather information about quadratic and exponential functions by interpreting various forms of expressions representing the functions. For example, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They relate their prior experience with transformations to that of building new functions from existing ones and recognize the effect of the transformations on the graphs. Formal work with complex numbers and more specialized functions—absolute value, step, and piecewise-defined, will occur in Algebra 2/Mathematics 3.

For further information on the Critical areas of focus for Algebra 1, please visit the Ohio Department of Education website at the following link: https://education.ohio.gov/getattachment/Topics/Learning-in-Ohio/Mathematics/Ohio-s-Learning-Standards-in-Mathematics/Transitioning-to-the-2017-Learning-Standards-in-Ma/GEOMETRY-CAF.pdf.aspx

RECOMMENDATIONS FOR ALGEBRA INSTRUCTION

The following are recommendations from the Institute of Educational Sciences on areas that educators can focus on to improve achievement in the Algebra classroom. For more detail on these recommendations, please visit the IES website at the following link:
https://ies.ed.gov/ncee/wwc/PracticeGuide/20

Recommendation 1: Use solved problems to engage students in analyzing algebraic reasoning and strategies. 1. Have students discuss solved problem structures and solutions to make connections among strategies and reasoning. 2. Select solved problems that reflect the lesson’s instructional aim, including problems that illustrate common errors. 3. Use whole-class discussions, small-group work, and independent practice activities to introduce, elaborate on, and practice working with solved problems.

Solved problem: An example that shows both the problem and the steps used to reach a solution to the problem. A solved problem can be pulled from student work or curricular materials, or it can be generated by the teacher. A solved problem is also referred to as a "worked example."

Sample solved problem:

Solve for x in this equation: $3^{4x+3} = 81$	$3^{4x+3} = 81$ $3^{4x+3} = 3^4$ $4x + 3 = 4$ $4x = 1$ $x = \frac{1}{4}$

Recommendation 2: Teach students to utilize the structure of algebraic representations. 1. Promote the use of language that reflects mathematical structure. 2. Encourage students to use reflective questioning to notice structure as they solve problems. 3. Teach students that different algebraic representations can convey different information about an algebra problem.

Example 2.1. Seeing structure in algebraic representations

Consider these three equations:

$$\mathbf{2}x + 8 = 14$$

$$\mathbf{2}(x + 1) + 8 = 14$$

$$\mathbf{2}(3x + 4) + 8 = 14$$

Though the equations appear to differ, they have similar structures: in all three equations,

2

multiplied by a quantity,

plus 8,

equals 14.

Paying attention to structure helps students make connections among problems, solution strategies, and representations that may initially appear different but are actually mathematically similar (Example 2.1). With an understanding of structure, students can focus on the mathematical similarities of problems that may appear to be different, which can simplify solving algebra problems. In particular, recognizing structure helps students understand the characteristics of ***algebraic expressions*** *and problems regardless of whether the problems are presented in symbolic, numeric, verbal, or graphic forms.*

Recommendation 3: Teach students to intentionally choose from alternative algebraic strategies when solving problems. 1. Teach students to recognize and generate strategies for solving problems. 2. Encourage students to articulate the reasoning behind their choice of strategy and the mathematical validity of their strategy when solving problems. 3. Have students evaluate and compare different strategies for solving problems.

Solution via conventional method	Solution via alternative method
Evaluate $2a + 4b - 7a + 2b - 8a$ if $a = 1$ and $b = 7$	
$2a + 4b - 7a + 2b - 8a$ $2(1) + 4(7) - 7(1) + 2(7) - 8(1)$ $2 + 28 - 7 + 14 - 8$ 29	$2a + 4b - 7a + 2b - 8a$ $-13a + 6b$ $-13(1) + 6(7)$ 29
Our restaurant bill, including tax but before tip, was $16.00. If we wanted to leave exactly 15% tip, how much money should we leave in total?	
$16.00 * 1.15 = x$ $x = \$18.40$	10% of $16.00 is $1.60, and half of $1.60 is $0.80, which totals $2.40, so the total bill with tip would be $16.00 + $2.40 or $18.40.
Solve for x: $3(x + 1) = 15$	
$3(x + 1) = 15$ $3x + 3 = 15$ $3x = 12$ $x = 4$	$3(x + 1) = 15$ $x + 1 = 5$ $x = 4$

GRADES 9-12 – ALGEBRA STANDARDS

SEEING INSTRUCTION IN EXPRESSIONS

- ***Interpret the structure of expressions.***

Standard

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, to factor 3x(x − 5) + 2(x − 5), students should recognize that the "x − 5" is common to both expressions being added, so it simplifies to (3x + 2) (x − 5); or see x4 − y4 as (x2)2 − (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 − y2) (x2 + y2).*

- ***Write expressions in equivalent forms to solve problems.***

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions for exponential functions. *For example, 8t can be written as 23t.*

(+) A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS

- ***Perform arithmetic operations on polynomials.***

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)
b. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)

- ***Understand the relationship between zeros and factors of polynomials.***

A.APR.2 Understand and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by *x − a* is *p (a).* In particular, *p (a) = 0* if and only if *(x – a)* is a factor of *p(x).*

A.APR.3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.

- ***Use polynomial identities to solve problems.***

A.APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

(+) A.APR.5 Know and apply the Binomial Theorem for the expansion of (*x* + *y*) *n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers. *For example by using coefficients determined for by Pascal's Triangle.* The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

● ***Rewrite rational expressions.***
Standard
A.APR.6 Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x)*, *b(x)*, *q(x)*, and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.
(+) A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
CREATING EQUATIONS ● ***Create equations that describe numbers or relationships.***
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★ a. Focus on applying linear and simple exponential expressions. (A1, M1) b. Focus on applying simple quadratic expressions. (A1, M2) c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ a. Focus on applying linear and simple exponential expressions. (A1, M1) b. Focus on applying simple quadratic expressions. (A1, M2) c. Extend to include more complicated function situations with the option to graph with technology. (A2, M3)
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ (A1, M1) a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★ a**.** Focus on formulas in which the variable of interest is linear or square. *For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle A = (π)r2 to highlight radius r.* (A1) b**.** Focus on formulas in which the variable of interest is linear. *For example, rearrange Ohm's law V = IR to highlight resistance R.* (M1) c. Focus on formulas in which the variable of interest is linear or square. *For example, rearrange the formula for the area of a circle A = (π)r2 to highlight radius r.* (M2) d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

G: Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

REASONING WITH EQUATIONS AND INEQUALITIES ● ***Understand solving equations as a process of reasoning and explain the reasoning.***
Standard
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
● ***Solve equations and inequalities in one variable.***
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring. c. (+) Derive the quadratic formula using the method of completing the square.
● ***Solve systems of equations.***
A.REI.5 Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A.REI.6 Solve systems of linear equations algebraically and graphically. a. Limit to pairs of linear equations in two variables. (A1, M1) b. Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)
A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*
(+) A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.
(+) A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
● ***Represent and solve equations and inequalities graphically.***
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

High School—Function

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph, e.g., the trace of a seismograph; by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

CONNECTIONS TO EXPRESSIONS, EQUATIONS, MODELING, AND COORDINATES.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

GRADES 9-12 – FUNCTIONS STANDARDS

INTERPRETING FUNCTIONS ● ***Understand the concept of a function, and use function notation.***
Standard
F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f(x)*.
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.
● ***Interpret functions that arise in applications in terms of the context.***
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★(A2, M3) a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2)
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2) c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)

● ***Analyze functions using different representations.***
Standard
F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★ a. Graph linear functions and indicate intercepts. (A1, M1) b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3) d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3) e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1) f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. (A2, M3) (+) g. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (+) h. Graph logarithmic functions, indicating intercepts and end behavior.
F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) **i.** Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1, M2) b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02) t, and y = (0.97) t and classify them as representing exponential growth or decay.* (A2, M3) **ii.** Focus on exponential functions evaluated at integer inputs. (A1, M2)
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3) a. Focus on linear and exponential functions. (M1) b. Focus on linear, quadratic, and exponential functions. (A1, M2)

G: Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

BUILDING FUNCTIONS ● ***Build a function that models a relationship between two quantities.***
Standard
F.BF.1 Write a function that describes a relationship between two quantities.★ a**.** Determine an explicit expression, a recursive process, or steps for calculation from context. **i.** Focus on linear and exponential functions. (A1, M1) **ii.** Focus on situations that exhibit quadratic or exponential relationships. (A1, M2) b**.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* (A2, M3) (+) **c.** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
● ***Build new functions from existing functions.***
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) a. Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)
F.BF.4 Find inverse functions. a. Informally determine the input of a function when the output is known. (A1, M1) (+) b**.** Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3) (+) c. Verify by composition that one function is the inverse of another. (A2, M3) (+) d. Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3) (+) e. Produce an invertible function from a non-invertible function by restricting the domain.
(+) F.BF.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
LINEAR, QUADRATIC, AND EXPONENTIAL MODELS ● ***Construct and compare linear, quadratic, and exponential models, and solve problems.***
F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★ a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

● ***Construct and compare linear, quadratic, and exponential models, and solve problems, continued.***
Standard
F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2)
F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.★
● ***Interpret expressions for functions in terms of the situation they model.***
F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★
TRIGONOMETRIC FUNCTIONS ● ***Extend the domain of trigonometric functions using the unit circle.***
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
(+) F.TF.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for *x*, where *x* is any real number.
(+) F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions**.**
● ***Model periodic phenomena with trigonometric functions.***
F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
(+) F.TF.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
(+) F.T.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★
● ***Prove and apply trigonometric identities.***
F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
(+) F.TF.9 Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes— as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non- right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

CONNECTIONS TO EQUATIONS

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

CRITICAL AREAS OF FOCUS FOR GEOMETRY

In Geometry, instructional time should focus on five critical areas:

CRITICAL AREA OF FOCUS #1 Applications of Probability Building on probability concepts that began in grade 7, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions related to real-world situations.

CRITICAL AREA OF FOCUS #2 Congruence, Proof and Constructions In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent or to have symmetries of itself, rotational or reflected. Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal and informal proof. Students prove theorems—using a variety of formats—and apply them when solving problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work. Students will extend prior experience with geometric shapes toward the development of a hierarchy of two-dimensional figures based on formal properties.

CRITICAL AREA OF FOCUS #3 Similarity, Proof, and Trigonometry Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use it as a familiar foundation for the development of informal and formal proofs, problem solving and applications to similarity in right triangles. This will assist in the further development of right triangle trigonometry, with particular attention to special right triangles, right triangles with one side and one acute angle given and the Pythagorean Theorem. Students apply geometric concepts to solve real-world, design and modeling problems.

CRITICAL AREA OF FOCUS #4 Connecting Algebra and Geometry Through Coordinates Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

CRITICAL AREA OF FOCUS #5 Circles With and Without Coordinates Students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. Students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done with systems of equations in the first course to determine intersections between lines and circles. Students model and solve real-world problems applying these geometric concepts.

For further details on the critical areas of focus, please visit the Ohio Department of Education website at the following link:
https://education.ohio.gov/getattachment/Topics/Learning-in-Ohio/Mathematics/Ohio-s-Learning-Standards-in-Mathematics/Transitioning-to-the-2017-Learning-Standards-in-Ma/GEOMETRY-CAF.pdf.aspx

GRADES 9-12 – GEOMETRY STANDARDS

CONGRUENCE ● ***Experiment with transformations in the plane.***
Standard
G.CO.1 Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.
G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.
G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.
G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
● ***Understand congruence in terms of rigid motions.***
G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
● ***Prove geometric theorems both formally and informally using a variety of methods.***
G.CO.9 Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
G.CO.10 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
G.CO.11 Prove and apply theorems about parallelograms. *Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

[G:] Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

● ***Make geometric constructions.***
Standard
G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
● ***Classify and analyze geometric figures.***
G.CO.14 Classify two-dimensional figures in a hierarchy based on properties.
SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY ● ***Understand similarity in terms of similarity transformations.***
G.SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor: a**.** A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b**.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
G.SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
● ***Prove and apply theorems both formally and informally involving similarity using a variety of methods.***
G.SRT.4 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.
● ***Define trigonometric ratios, and solve problems involving right triangles.***
G.SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
G.SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8 Solve problems involving right triangles.★ a. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2) b. (+) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ (A2, M3)

[G:] Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

● ***Apply trigonometry to general triangles.***
Standard
(+) **G.SRT.9** Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
(+) **G.SRT.10** Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.
(+) **G.SRT.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.
CIRCLES ● ***Understand and apply theorems about circles.***
G.C.1 Prove that all circles are similar using transformational arguments.
G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
G.C.3 Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.
(+) **G.C.4** Construct a tangent line from a point outside a given circle to the circle.
● ***Find arc lengths and areas of sectors of circles.***
G.C.5 Find arc lengths and areas of sectors of circles. a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. b. Derive the formula for the area of a sector, and use it to solve problems.
G.C.6 Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)
EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS ● ***Translate between the geometric description and the equation for a conic section.***
G.GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
(+) G.GPE.2 Derive the equation of a parabola given a focus and directrix.
(+) G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

● ***Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.***
Standard
G.GPE.4 Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. *For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle.* (G, M2)
G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.
G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★
GEOMETRIC MEASUREMENT AND DIMENSION ● ***Explain volume formulas, and use them to solve problems.***
G.GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
(+) **G.GMD.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
G.GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★
● ***Visualize relationships between two-dimensional and three-dimensional objects.***
G.GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
● ***Understand the relationships between lengths, area, and volumes.***
G.GMD.5 Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.
G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of *k*, the effect on lengths, areas, and volumes is that they are multiplied by *k*, *k2*, and *k3*, respectively.
MODELING WITH GEOMETRY ● ***Apply geometric concepts in modeling situations.***
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★
G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★
G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

High School—Statistics and Probability

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

CONNECTIONS TO FUNCTIONS AND MODELING

Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

GRADES 9-12 – STATISTICS AND PROBABILITY STANDARDS

INTERPRETING CATEGORICAL AND QUANTITATIVE DATA ● ***Summarize, represent, and interpret data on a single count or measurement variable.***
Standard
S.ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.★
S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. ★
S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★
● ***Summarize, represent, and interpret data on two categorical and quantitative variables.***
S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★
S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3) b. Informally assess the fit of a function by discussing residuals. (A2, M3) c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)
● ***Interpret linear models.***
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★
S.ID.9 Distinguish between correlation and causation.★
MAKING INFERENCES AND JUSTIFYING CONCLUSIONS ● ***Understand and evaluate random processes underlying statistical experiments.***
S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★
S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★

G: Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

● ***Make inferences and justify conclusions from sample surveys, experiments, and observational studies.***
Standard
S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★
S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★
S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.★
S.IC.6 Evaluate reports based on data.★
CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY ● ***Understand independence and conditional probability, and use them to interpret data.***
S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★
S.CP.2 Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★
S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★
S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★
S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★
● ***Use the rules of probability to compute probabilities of compound events in a uniform probability model.***
S.CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.★
S.CP.7 Apply the Addition Rule, P (A or B) = P (A) + P (B) − P (A and B), and interpret the answer in terms of the model.★
(+) S.CP.8 Apply the general Multiplication Rule in a uniform probability model[G], P (A and B) = P (A) ·P (B|A) = P (B) ·P (A|B), and interpret the answer in terms of the model.★
(+) S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.★

G: Refer to official definition from Ohio’s Learning Standards for Mathematics document glossary.

USING PROBABILITY TO MAKE DECISIONS ● ***Calculate expected values, and use them to solve problems.***
Standard
(+) S.MD.1 Define a random variable[G] for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution[G] using the same graphical displays as for data distributions.★
(+) S.MD.2 Calculate the expected value[G] of a random variable; interpret it as the mean of the probability distribution.★
(+) **S.MD.3** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*★
(+) **S.MD.4** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* ★
● ***Use probability to evaluate outcomes of decisions.***
(+) S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★ a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
(+) S.MD.6 Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★
(+) S.MD.7 Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

[G:] Refer to official definition from Ohio's Learning Standards for Mathematics document glossary.

PERFORMANCE LEVEL DESCRIPTORS

Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – GRADE 6

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 6 Mathematics. A student whose performance falls within the **Proficient Level** has a **consistent ability** to divide fractions by fractions and demonstrate understanding of negative rational numbers, use ratio reasoning to solve problems, use expressions with variables to represent and solve problems, and use visual displays to solve problems involving the coordinate plane, to find measures of two- and three-dimensional figures, and to summarize of data.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Grade 6 Mathematics. A student at this level has a **superior ability** to divide fractions by fractions and demonstrate understanding of negative rational numbers, use ratio reasoning to solve problems, use expressions with variables to represent and solve problems, and use visual displays to solve problems involving the coordinate plane, to find measures of two- and three-dimensional figures, and to summarize distributions of data.	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Grade 6 Mathematics. A student at this level has a **sophisticated ability** to divide fractions by fractions and demonstrate understanding of negative rational numbers, use ratio reasoning to solve problems, use expressions with variables to represent and solve problems, and use visual displays to solve problems involving the coordinate plane, to find measures of two- and three-dimensional figures, and to summarize distributions of data.
A student at the **Proficient Level** can: • Write a ratio to describe a relationship between two quantities; • Understand the concept of unit rates; • Find a percent of a quantity as a rate per 100; • Solve a wide variety of routine problems involving ratios and rates; • Use ratio tables to solve routine real-world problems; • Solve routine mathematical and real-world unit rate problems (including unit pricing and constant speed);	A student at the **Accelerated Level** can: • Select appropriate representations and strategies to solve mathematical and real-world ratio and rate problems; • Solve a wide variety of problems involving ratios and rates; • Apply ratio reasoning to convert measurement units within the same system; • Write expressions and equations that correspond to given situations; • Evaluate complex algebraic expressions including exponents;	A student at the **Advanced Level** can: • Select efficient representations and strategies to solve mathematical and real-world ratio and rate problems; • Solve a wide variety of real-world problems involving ratios and rates, including where a ratio is associated with a rate; • Solve a variety of non-routine problems requiring conversion of measurement units. • Write expressions and equations for complex mathematical and real-world situations;

• Convert measurement units within the same system using ratio reasoning; • Write and evaluate numerical expressions including those with whole number exponents; • Understand the use of variables in simple mathematical expressions; • Write expressions and equations that correspond to given routine situations; • Evaluate algebraic expressions; • Apply the understanding of equivalent expressions to identify equivalent expressions; • Write and solve one-step equations with positive integer coefficients; • Write and graph solutions to inequalities on a number line; • Write a one-variable equation to express one quantity in terms of the other quantity; • Divide a fraction by a fraction using a visual model; • Find the greatest common factor of two numbers less than or equal to 100; • Find the least common multiple of two whole numbers less than or equal to 12; • Add, subtract, multiply, and divide multi-digit decimals; • Use number lines to compare and order positive and negative numbers; • Represent real-world quantities with positive and negative numbers; • Locate points and ordered pairs in all four quadrants of the coordinate plane (understand the signs in ordered pairs); • Calculate median, mean, and range; • Display numerical data using number lines, dot plots, histograms, and box plots; • Find areas of polygons by decomposing them into rectangles and triangles;	• Apply the properties of operations to write equivalent expressions; • Write and solve equations with positive rational coefficients; • Given a situation, write an inequality and graph solutions on a number line; • Use tables and graphs to analyze the relationship between dependent and independent variables and relate these to the equation; • Divide a fraction by a fraction using visual models and equations; • Use least common multiples and greatest common factors to solve routine real-world problems; • Add, subtract, multiply, and divide multi-digit decimals to solve real-world problems; • Use number lines to compare and order positive and negative numbers; • Represent real-world quantities with positive and negative numbers; • Locate points and ordered pairs in all four quadrants of the coordinate plane (understand the signs in ordered pairs); • Calculate interquartile range; • Choose the correct measure of center relating to a certain context; • Describe and summarize numerical distributions (data sets) by identifying clusters, peaks, gaps, and symmetry, in relationship to the context in which the data were collected; • Display and interpret numerical data using number lines, dot plots, histograms, and box plots; • Solve mathematical problems by finding the area of a two-dimensional shape composed of rectangles and triangles;	• Explain why two expressions are equivalent using precise mathematical language; • Given a complex situation, write an inequality and graph solutions on a number line; • Analyze the relationship between dependent and independent variables represented in tables and graphs, and then write an appropriate equation; • Divide a fraction by a fraction; • Efficiently use least common multiples and greatest common factors to solve real-world problems; • Write, interpret, and explain statements of order in real-world contexts; • Represent real-world quantities with positive and negative numbers; • Graph ordered pairs in all four quadrants of the coordinate plane; • Choose, calculate, and justify the correct measure of center relating to a certain context; • Describe numerical distributions (data sets) by identifying clusters, peaks, gaps, and symmetry, in relationship to the context in which the data were collected; • Solve non-routine mathematical problems by finding the area of a two-dimensional shape composed of rectangles and triangles; • Solve real-world problems by finding the volumes of rectangular prisms with fractional edge lengths; • Solve real-world and mathematical problems by graphing points and/or polygons in the coordinate plane.

• Find volumes of rectangular prisms with fractional edge lengths; • Draw polygons in the coordinate plane; • Solve routine real-world and mathematical problems by graphing points in the first quadrant; • Use nets to find surface areas of rectangular and triangular prisms and pyramids.	• Solve mathematical problems by finding the volumes of rectangular prisms with fractional edge lengths; • Draw polygons in the coordinate plane and find lengths of horizontal and vertical sides to solve real-world problems; • Solve real-world and mathematical problems by graphing points in the first quadrant.	

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates partial command of Ohio's Learning Standards for Grade 6 Mathematics. A student at this level has a **general ability** to divide fractions by fractions and demonstrate understanding of negative rational numbers, use ratio reasoning to solve problems, use expressions with variables to represent and solve problems, and use visual displays to solve problems involving the coordinate plane, to find measures of two- and three-dimensional figures, and to summarize distributions of data.

A student whose performance falls within the **Basic Level** typically can**:**

• Carry out routine procedures;
• Solve simple problems using visual representations;
• Compute accurately some grade level numbers and operations;
• Recall and recognize some grade level mathematical concepts, terms, and properties, and use more previous grade level mathematical concepts, terms, and properties.

A student at the **Basic Level** can:

• Write a ratio to describe a familiar relationship between two quantities;
• Find missing values in tables of equivalent ratios;
• Solve familiar straightforward unit rate problems;
• Find a percent of a quantity as a rate per 100 using 100 grids;
• Solve routine straightforward problems involving ratios;
• Solve routine straightforward real-world unit rate problems (including unit pricing);
• Complete a table of familiar measurement unit conversions within the same system;
• Write and evaluate numerical expressions with up to two operations including those with exponents of 2 and 3;
• Understand the use of variables in simple mathematical expressions;

• Identify one- and two-step expressions and equations that correspond to given familiar situations;
• Evaluate algebraic expressions with up to two operations;
• Identify up to two-step equivalent expressions;
• Solve one-step equations with positive integer coefficients;
• Write a one-variable equation to express one quantity in terms of the other quantity.

• Interpret a visual model for division of a fraction by a fraction;
• Divide multi-digit whole numbers;
• Add and subtract multi-digit decimal numbers;
• Divide multi-digit decimals by whole number divisors;
• Find common factors of two numbers less than or equal to 100;
• Find common multiples of two numbers less than or equal to 12;
• Use positive and negative numbers to represent quantities in real-world contexts;
• Find and position positive and negative rational numbers on a horizontal or vertical number line and on a coordinate plane;
• Use number lines to compare and order positive and negative numbers;
• Represent real-world quantities with positive and negative numbers;
• Locate points in all four quadrants of the coordinate plane;
• Find the median of an even number of whole number data points; find the mean of whole number data points;
• Display numerical data using number lines and dot plots;
• Find areas of polygons with whole number side lengths by decomposing them into rectangles and triangles;
• Find volumes of rectangular prisms with whole number side lengths;
• Draw polygons in one quadrant of the coordinate plane;
• Solve routine real-world and mathematical problems by graphing points in the first quadrant;
• Given nets, find surface areas of rectangular prisms with whole number side lengths.

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Grade 6 Mathematics. A student at this level has an **emerging ability** to divide fractions by fractions and demonstrate understanding of negative rational numbers, use ratio reasoning to solve problems, use expressions with variables to represent and solve problems, and use visual displays to solve problems involving the coordinate plane, to find measures of two- and three-dimensional figures, and to summarize distributions of data.

A student whose performance lies within the **Limited Level** typically can:

• Carry out some routine procedures to solve straightforward one-step problems;
• Recognize solutions to straightforward problems involving some simple computation;
• Compute a few numbers accurately;
• Recognize a few grade level mathematical concepts, terms, and properties, and use previous grade level mathematical concepts, terms, and properties.

A student at the **Limited Level** can:

• Write a ratio to describe a familiar relationship between two quantities using given information;
• Find the percent of a quantity using 100 grids;
• Use models to solve simple problems involving ratios;
• Complete simple ratio tables;
• Solve simple routine unit rate problems;
• Plot pairs of positive values on the coordinate plane.
• Evaluate numerical expressions with two operations;
• Understand the use of variables in simple mathematical expressions;
• Identify expressions and equations that correspond to given routine situations;
• Evaluate one-step expressions;
• Apply the understanding of equivalent expressions to identify equivalent expressions;
• Solve simple one-step equations involving addition and subtraction;
• Write a one-variable equation to express one quantity in terms of the other quantity;
• Use substitution to determine whether a given number makes an equation true.
• Divide simple multi-digit whole numbers;
• Recognize a visual model for division of a fraction by a fraction;
• Add, subtract, and multiply multi-digit whole numbers and decimals to hundredths using strategies and algorithms;
• Can identify or locate a positive or negative whole number on a number line.
• Identify the median of an odd number of whole number data points;
• Find areas of right triangles using grid paper;
• Find the volume of rectangular prisms with whole number sides;
• Draw polygons in the coordinate plane given coordinates in the first quadrant;
• Display simple numerical data using number lines and dot plots.

PCSD

Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – GRADE 7

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 7 Mathematics. A student at this level has a consistent ability to work with expressions and linear equations, solve problems involving scale drawings, and work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Grade 7 Mathematics. A student at this level has a superior ability to work with expressions and linear equations, solve problems involving scale drawings, and work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Grade 7 Mathematics. A student at this level has a sophisticated ability to work with expressions and linear equations, solve problems involving scale drawings, and work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.
A student at the **Proficient Level** can: • Compute a unit rate of two rational numbers where the unit rate is not explicitly requested; • Represent proportional relationships in various formats; • Use proportional relationships to solve routine real-world and mathematical ratio and percent problems with multiple steps; • Solve mathematical problems using the four operations on simple rational numbers; • Convert from fractions to decimals without technology; • Apply properties of operations to factor and expand linear expressions with simple rational coefficients;	A student at the **Accelerated Level** can: • Compare unit rates in a real-world context; • Use different representations of proportional relationships to solve real-world problems; • Apply proportional relationships to routine real-world and mathematical ratio and percent problems with multiple steps; • Solve mathematical problems using the four operations on rational numbers; • Apply properties of operations to factor and expand linear expressions with rational coefficients; • Understand that rewriting an expression can show how quantities are related in familiar problem-solving contexts;	A student at the **Advanced Level** can: • Analyze a graph of a proportional relationship in order to explain what the points (x, y) and (1, r) represent, where r is the unit rate, and use this to solve problems; • Apply proportional relationships to non-routine real-world and mathematical ratio and percent problems with multiple steps; • Interpret products and quotients of rational numbers in real-world contexts; • Apply properties of operations to factor and expand linear expressions with complex rational coefficients; • Understand that rewriting an expression can show how quantities are related in an unfamiliar problem-solving context;

• Use variables to create and solve simple equations and inequalities that model word problems; • Solve problems involving scale drawings of geometric figures, including computing actual areas from a scale drawing and represent proportional relationships among similar figures; • Using technology or math tools, determine whether a set of any three given angle or side length measures can result in a unique triangle, more than one triangle, or no triangles at all and construct quadrilaterals with given conditions; • Identify the two-dimensional figures that result from routine slices of prisms and pyramids; • Use supplementary, complementary, vertical, and adjacent angles to solve one- or two-step problems with angle measurements expressed as variables in degrees; • Solve problems involving the area of two-dimensional objects composed of triangles, quadrilaterals, and polygons; • Calculate the area and circumference of a circle in real-world and mathematical problems; • Solve routine real-world and mathematical problems involving the surface area and volume of three-dimensional objects composed of cubes and right prisms. • Describe a sample of a given population.	• Construct equations and inequalities with a variable to solve routine problems; • Solve real-world problems involving similar figures; • Identify the two-dimensional figures that result from non-routine slices of prisms and pyramids; • Use supplementary, complementary, vertical, and adjacent angles to solve multi-step problems with angle measurements expressed as variables in degrees. • Given the circumference of a circle, determine its area; • Solve real-world and mathematical problems involving the surface area three- dimensional objects composed of triangles and rectangles; • Use measures of variability for numerical data from random samples to draw informal comparative inferences about two populations; • Find probabilities of compound events in a real-world context; • Use example situations to explain the differences between theoretical and experimental probabilities.	• Construct equations and inequalities with more than one variable to solve non- routine problems; • Use variables to represent and reason with quantities in real-world and mathematical situations; • Reproduce scale drawings at a different scale to solve real-world problems; • Solve problems using formulas for the area and circumference of a circle; • Informally describe the relationship between the two measures; • Solve complex problems involving the surface area and volume of three- dimensional figures with polygonal faces; • Assess the degree of visual overlap of two numerical data distributions with similar variability; • Use measures of variability for numerical data from random samples to draw informal comparative inferences about multiple populations; • Explain why events are likely or unlikely and use this explanation to make predictions; • Develop a probability model and use it to find probabilities of events; • Compare theoretical probabilities (from a model) to observed frequencies (experimental); explain possible sources of the discrepancy between the two measures.

• Solve problems involving the area of two-dimensional objects composed of triangles, quadrilaterals, and polygons; • Calculate the area and circumference of a circle in real-world and mathematical problems; • Solve routine real-world and mathematical problems involving the surface area and volume of three-dimensional objects composed of cubes and right prisms. • Describe a sample of a given population. • Understand that a probability near 0 indicates an unlikely event, a probability near ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event; • Compare theoretical and experimental results from a probability experiment.		

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates partial command of Ohio's Learning Standards for Grade 7 Mathematics. A student at this level has a general ability to work with expressions and linear equations, solve problems involving scale drawings, and work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.

A student whose performance falls within the **Basic Level** typically can:

- Carry out routine procedures;
- Solve simple problems using visual representations;
- Compute accurately some grade level numbers and operations;
- Recall and recognize some grade level mathematical concepts, terms and properties, and use more previous grade level mathematical concepts, terms and properties.

A student at the **Basic Level** can:

- Compute a unit rate of two familiar rational numbers where the unit rate is explicitly requested;
- Find the whole number constant of proportionality in relationships presented in basic familiar contexts;
- Solve a one-step, straightforward real-world ratio or percent problem.
- Add, subtract, multiply and divide integers;
- Convert between familiar fractions and decimals;
- Apply properties of operations to factor and expand linear expressions with positive integer coefficients;
- Solve two-step equations with integer coefficients;
- Solve simple inequalities with positive integer coefficients;
- Determine a scale from scale drawings of geometric figures and compute an actual length given a measurement in a scale drawing and the scale;
- Draw geometric shapes with given conditions;
- Determine whether a set of any three given angle or side length measurements can result in a triangle or whether a quadrilateral could be represented by given angles and/or side lengths;
- Use supplementary, complementary, vertical, or adjacent angles to solve problems with angles expressed as numerical measurements;
- Calculate the area of quadrilaterals and polygons;
- Calculate the volume of right rectangular prisms;
- Calculate the circumference of a circle in mathematical problems;
- Explain whether a sample is random;
- Use measures of center to draw comparisons about two different populations;
- Find probabilities in straightforward situations;
- Use measures of center to draw comparisons about two different populations;
- Find probabilities in straightforward situations.

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Grade 7 Mathematics. A student at this level has an emerging ability to work with expressions and linear equations, solve problems involving scale drawings, and work with two- and three-dimensional shapes to solve problems involving area, surface area, and volume.

A student whose performance falls within the **Limited Level** typically can:

- Carry out some routine procedures to solve straightforward one-step problems;
- Recognize solutions to some simple computation, straightforward problems;
- Compute accurately a few grade level numbers and operations;
- Recognize a few grade level mathematical concepts, terms and properties, and use previous grade level mathematical concepts, terms and properties.

A student at the **Limited Level** can:

- Compute a unit rate of two whole numbers where the unit rate is explicitly requested;
- Identify proportional relationships presented in familiar contexts;
- Solve a one-step, straightforward ratio or percent problem;
- Model addition and subtraction of simple rational numbers on the number line;
- Recognize the additive inverse property;
- Recognize simple equivalent expressions;
- Solve simple equations;
- Identify a solution of an inequality;
- Recognize simple geometric shapes based on given conditions;
- Classify pairs of angles;
- Identify the parts of a circle;
- Calculate the area of triangles and rectangles;
- Calculate the volume of cubes;
- Determine whether a sample is random;
- Use the mean to compare and draw inferences about two different populations;
- Understand that probabilities are numbers between 0 and 1.

PCSD

Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – GRADE 8

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Grade 8 Mathematics. A student at this level has a consistent ability to formulate and reason about expressions and equations, use functions to describe quantitative relationships, and analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and to understand and apply the Pythagorean Theorem.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Grade 8 Mathematics. A student at this level has a superior ability to formulate and reason about expressions and equations, use functions to describe quantitative relationships, and analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and to understand and apply the Pythagorean Theorem.	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Grade 8 Mathematics. A student at this level has a sophisticated ability to formulate and reason about expressions and equations, use functions to describe quantitative relationships, and analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and to understand and apply the Pythagorean Theorem.
A student at the **Proficient Level** can: • Graph proportional relationships, interpreting the unit rate as the slope and compare two different proportional relationships using different representations; • Solve routine multi-step linear equations with rational coefficients and variables on both sides and provide examples of equations that have one solution, infinitely many solutions, or no solutions; • Find or estimate the solution to a system of linear equations by graphing; • Describe patterns in scatterplots for routine contexts, such as: clustering, outliers, positive or negative association, linear association, and/or nonlinear association;	A student at the **Accelerated Level** can: • Apply understanding of slope to solve routine problems graphically and algebraically; • Strategically choose and use procedures to solve linear equations in one variable; • Justify why an equation has one solution, infinitely many solutions, or no solution; • Use the graph of a system of linear equations to represent, analyze and solve a variety of problems; • Compare more than one trend line for the same scatter plot; • Create and use a linear model based on a set of bivariate data to solve a problem in a routine context; • Justify whether two functions represented in different ways are equivalent or not by comparing their properties;	A student at the **Advanced Level** can: • Apply understanding of slope to solve non-routine problems graphically and algebraically; • Strategically and efficiently use graphs of systems of linear equations to represent, analyze and solve a variety of problems; • Compare more than one trend line for the same scatter plot and justify the best one; • Construct and interpret scatter plots for bivariate measurements data to investigate patterns of association between two quantities; • Create and use a linear model based on a set of bivariate data to solve problems in a variety of non-routine contexts; • Interpret, describe and compare relative frequencies to identify patterns of association in given contexts;

• Interpret and describe relative frequencies for possible associations from a two-way table representing a routine situation; • Complete a table to show a relation that is or is not a function; • Compare properties (i.e. slope, y-intercept, values) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or verbal descriptions); • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values; • Construct a function to model a linear relationship between two quantities; • Describe a sequence of rigid transformations between two congruent figures; • Recognize that a dilation produces a similar figure; • Understand and explain a proof of the Pythagorean Theorem and its converse; • Apply the Pythagorean Theorem to real-world situations that can be modeled in two dimensions to determine unknown side lengths; • Determine missing angle measures in triangles with exterior angles and/or angles formed by parallel lines cut by a transversal; • Solve real-world and mathematical problems involving the volumes of cones, cylinders and spheres; • Identify rational and irrational numbers and convert less familiar rational numbers (repeating decimals) to fraction form; • Place irrational numbers on a number line;	• Explain why a dilation produces a similar figure and that rigid transformations maintain angle measure and side lengths; • Give an informal argument that a triangle can only have one 90 angle; • Understand and explain the proof of the Pythagorean Theorem and its converse in multiple ways; • Apply the Pythagorean Theorem in multi-step mathematical and real-world problems in two and three dimensions; • Solve real-world and mathematical problems involving the volume of a composite solid including a cone, cylinder or sphere. • Place irrational numbers on a number line in an abstract setting using variables; • Use square root and cube root symbols to represent solutions to real-world problems resulting from equations of the form $x^2 = p$ and $x^3 = p$; • Solve problems involving the conversion between decimal notation and scientific notation and the comparison of numbers written in different notations.	• Explain why a function is linear or nonlinear; • Interpret qualitative features of a function in a context; • Strategically and efficiently choose different ways to represent functions in solving a variety of problems; • Justify why two figures are congruent and/or similar; • Solve a variety of real-world and mathematical problems involving the angles in triangles and those formed by when parallel lines are cut by a transversal, and give informal arguments; • Informally explain the derivation of the formulas for cones, cylinders, and spheres; • Notice and explain the patterns that exist when writing rational numbers (repeating decimals) as fractions; • Explain how to get more precise approximations of square roots; • Use properties of integer exponents to order or evaluate multiple numerical expressions with integer exponents; • Explain how square roots and cube roots relate to each other and to their radicands; • Calculate and interpret values written in scientific notation within new and unfamiliar contexts.

• Apply the properties of integer exponents to solve mathematical problems; • Use square root and cube root symbols to represent solutions of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number; • Express how many times a number written as an integer power of 10 is than another number written as an integer power of 10; • Solve routine problems that require performing operations with numbers expressed in scientific notation, including numbers written in both decimal and scientific notation and interprets scientific notation that has been generated by technology.		

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates partial command of Ohio's Learning Standards for Grade 8 Mathematics. A student at this level has a general ability to formulate and reason about expressions and equations, use functions to describe quantitative relationships, and analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and to understand and apply the Pythagorean Theorem.

A student whose performance falls within the **Basic Level** typically can:

- Carry out routine procedures;
- Solve simple problems using visual representations;
- Compute accurately some grade level numbers and operations;
- Recall and recognize some grade level mathematical concepts, terms and properties, and use more previous grade level mathematical concepts, terms and properties.

A student at the **Basic Level** can:

- Graph proportional relationships, interpreting the unit rate as the slope and compare two different proportional relationships using the same representation;
- Solve straightforward multi-step linear equations with rational coefficients;
- Solve a system of simple linear equations by inspection and graphically;
- Construct a scatter plot and describe the pattern as positive, negative or no relationship;
- Draw a straight line on a scatter plot that closely fits the data points;
- Construct a two-way table of categorical data;
- Given tables of ordered pairs, determine if the relation is a function;
- Compare properties (i.e. slope, y-intercept, values) of two functions each represented in the same way (algebraically, graphically, or verbal descriptions);
- Create an image of a geometric figure using a reflection over an axis and/or multiple translations;
- Create dilations of figures by a given whole number scale factor;
- Calculate unknown side lengths using the Pythagorean Theorem given a picture of a right triangle;
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system with the right triangle drawn;
- Find the volume of a cone, cylinder or sphere given the height and/or radius;
- Identify between which two whole number values a square root of a non-square number is located;
- Apply the properties of natural number exponents to solve simple mathematical problems;
- Calculate the cube root of small perfect cubes;
- Use scientific notation to represent and compare very large and very small quantities.

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Grade 8 Mathematics. A student at this level has an emerging ability to formulate and reason about expressions and equations, use functions to describe quantitative relationships, and analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and to understand and apply the Pythagorean Theorem.

A student whose performance falls within the **Limited Level** typically can:

• Carry out some routine procedures to solve straightforward one-step problems;
• Recognize solutions to some simple computation, straightforward problems;
• Compute accurately a few grade level numbers and operations;
• Recognize a few grade level mathematical concepts, terms and properties, and use previous grade level mathematical concepts, terms and properties.

A student at the **Limited Level** can:

• Graph proportional relationships, interpreting the unit rate as the slope;
• Determine the slope of a line given a graph;
• Solve straightforward one or two step linear equations with integer coefficients;
• Construct a scatter plot;
• Recognize a straight line can be used to describe a linear association on a scatter plot;
• Identify the slope and y-intercept of a linear model on a scatter plot;
• Identify whether a relation is a function from a graph or a mapping;
• Compare properties (i.e. slope, y-intercept, values) of two functions in a graph;
• Given a straightforward qualitative description of a functional relationship between two quantities, sketch a graph;
• Create a single translation of a geometric figure;
• Identify two congruent figures;
• Identify if two figures are related by a dilation, translation, rotation, or reflection;
• Identify pairs of equivalent angles when parallel lines are cut by a transversal;
• Use the Pythagorean Theorem to calculate the hypotenuse in mathematical problems;
• Identify square roots of non-square numbers and pi as irrational numbers;
• Use the properties of natural number exponents to generate equivalent numerical expressions;
• Evaluate square roots of small perfect squares;
• Use numbers expressed in the form of a single digit times an integer power of 10 to estimate

Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – ALGEBRA

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Algebra. A student at this level has a consistent ability to demonstrate reasoning with numbers, quantities, expressions, and equations to solve problems, to write and analyze functions to model and solve problems, and to summarize and interpret data on one or two variables.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Algebra. A student at this level has a superior ability to demonstrate reasoning with numbers, quantities, expressions, and equations to solve problems, to write and analyze functions to model and solve problems, and to summarize and interpret data on one or two variables.	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Algebra. A student at this level has a sophisticated ability to demonstrate reasoning with numbers, quantities, expressions, and equations to solve problems, to write and analyze functions to model and solve problems, and to summarize and interpret data on one or two variables.
A student at the **Proficient Level** can: • Choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays; • Interpret parts of a simple exponential expression, such as terms, factors, coefficients, bases, and exponents in terms of its context; • Recognize the structure of a quadratic expression to identify ways to rewrite it to better represent the purpose; • Factor a quadratic expression to reveal the zeros of the function it defines; • Create exponential equations in one or two variables and use them to solve routine problems; • Graph equations on coordinate axes with appropriate labels and scales;	A student at the **Accelerated Level** can: • Interpret linear expressions by viewing one or more of their parts as a single entity in respect to the context; • Use the structure of an exponential expression to identify ways to rewrite it; • Create quadratic and exponential equations and inequalities in one or two variables and use them to solve routine problems; • Solve multi-step linear equations and inequalities with rational coefficients in one variable situations; • Solve multistep linear equations with coefficients represented by letters, including formulas, which could include factoring or distributive property;	A student at the **Advanced Level** can: • Accurately multiply polynomials of any number of terms using rules of exponents; • Interpret exponential expressions by viewing one or more of their parts as a single entity in relationship to the context; • Create quadratic and exponential equations and inequalities in one or two variables and use them to accurately solve routine and non-routine problems; • Find and interpret solutions as viable or non-viable options in a modeling context; • Construct a viable argument to justify a solution method for a quadratic equation; • Choose an appropriate method to solve a quadratic equation, according to the initial form of the equation, which could include

• Solve multi-step linear equations and inequalities with integer coefficients in one variable situations; • Solve multistep linear equations with coefficients represented by letters, including formulas; • Select a viable argument to justify a solution method for a simple linear equation; • Solve quadratic equations with integer coefficients and constants by factoring or quadratic formula, where solutions may be rational; • Solve quadratic equations with integer coefficients and constants by completing the square where *a* = 1; • Add and subtract polynomials and multiply monomials by polynomials; • Multiply binomials; • Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line; • Solve a system of linear inequalities in two variables graphically; • For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities; sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; • Relate the domain of a function to its graph;	• Construct a viable argument to justify a solution method for a system of linear equations; • Solve quadratic equations with rational coefficients and constants by factoring or completing the square; • Solve a system consisting of a linear equation and a quadratic equation in two variables graphically, and algebraically in simpler cases; • Multiply binomials by trinomials; • Find the approximate solutions of the equation f(x) = g(x) by finding the x-coordinates where the graphs of the equations y = f(x) and y = g(x) intersect, including cases where f(x) and/or g(x) are linear or exponential; • Create an explicit function to define an arithmetic sequence; • For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: end behavior; • Relate the domain of a function to the quantitative relationship it describes; • Graph simple exponential functions, showing end behavior; • Interpret zeros, extreme values, and symmetry of the graph of a quadratic function in terms of a context; • Use the process of completing the square in a quadratic function, where *a* = 1, to show extreme values and symmetry of the graph; • Use statistics appropriate to the shape of the data distribution to compare spread	simplifying initial expressions and with possible complex solutions; • Accurately and efficiently solve a system consisting of a linear equation and a quadratic equation in two variables algebraically; • Interpret statements that use function notation in terms of a context; • Create an explicit function to define a geometric sequence; • Recognize a recursive function that defines a sequence from a context; • Interpret the parameters in a linear or exponential function in terms of a context; • Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers);

• Recognize the difference between linear and exponential situations from real-world contexts or a variety of representations; • Graph quadratic functions and show intercepts, maxima, and minima; • Given two functions represented in different ways, (algebraically, graphically, numerically in tables, or by verbal descriptions), compare the properties of the two functions; • Use statistics appropriate to the shape of the data distribution to compare center (median, mean) of two or more different data sets; • Summarize categorical data for two categories in two-way frequency tables; • Interpret joint relative frequencies in the context of the data; • Fit a linear function for a scatter plot that suggests a linear association; • Compute (using technology) the correlation coefficient of a linear fit.	(interquartile range, standard deviation) of two or more different data sets; • Interpret conditional relative frequencies in the context of the data; • Find and use linear models to solve problems in the context of the data; • Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates partial command of Ohio's Learning Standards for Algebra. A student at this level has a general ability to demonstrate reasoning with numbers, quantities, expressions, and equations to solve problems, to write and analyze functions to model and solve problems, and to summarize and interpret data on one or two variables.

A student whose performance falls within the **Basic Level** typically can**:**

- Carry out routine procedures;
- Solve simple problems using visual representations;
- Compute accurately some grade level numbers and operations;
- Recall and recognize some grade level mathematical concepts, terms and properties, and use more previous grade level mathematical concepts, terms and properties.

A student at the **Basic Level** can:

- Choose and interpret units in formulas;
- Given a situation, context, or problem, students will identify, and use appropriate quantities for representing the situation;
- Identify parts of simple linear expressions in terms of the context the quantity represents: terms, factors and coefficients;
- Create linear equations in two variables and inequalities in one variable and use them to solve simple routine problems;
- Evaluate given possible solutions as viable or non-viable options in a modeling context;
- Solve simple linear equations and inequalities with integer coefficients in one variable situations;
- Solve one-step linear equations with coefficients represented by letters, including formulas;
- Solve routine quadratic equations with integer solutions;
- Add and subtract polynomials and multiply polynomials by constants, both supported by manipulatives or visual models;
- Solve simple systems of two linear equations in two variables exactly algebraically;
- Graph the solutions to a linear inequality in two variables as a half-plane;
- Understand that the graph of a function f is the graph of the equation $y = f(x)$;
- Given a graph of a function that models a linear relationship between two quantities, interpret key features: intercepts; intervals where the function is increasing, decreasing, positive, or negative;
- Recognize the difference between a linear and exponential situation represented by a graph or equation;
- Represent given data in a different statistical model;
- Interpret key features of a scatterplot (linear or nonlinear, correlation).

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Algebra. A student at this level has an emerging ability to demonstrate reasoning with numbers, quantities, expressions, and equations to solve problems, to write and analyze functions to model and solve problems, and to summarize and interpret data on one or two variables.

A student whose performance lies within the **Limited Level** typically can:

- Carry out some routine procedures to solve straightforward one-step problems;
- Recognize solutions to some simple computation, straightforward problems;
- Compute accurately a few grade level numbers and operations;
- Recognize a few grade level mathematical concepts, terms and properties, and use previous grade level mathematical concepts, terms and properties.

A student at the **Limited Level** can:

- Identify units in familiar formulas involving whole numbers;
- Identify parts of simple linear expressions: terms, factors and coefficients;
- Solve simple linear equations with integer coefficients and inequalities with whole number coefficients in one variable situations, with integer constants and whole number solutions;
- Solve linear equations in two variables to describe a familiar situation using whole numbers supported by algebra manipulatives or diagrams;
- Find square roots of perfect squares;
- Use algebra manipulatives or diagrams and the relationship of polynomials to whole numbers to add and subtract polynomials with like terms;
- Given a straightforward linear relationship in context, write a function;
- Given a graph of a simple function modeling a linear relationship between two quantities, determine where the function is increasing, decreasing, positive, or negative;
- Graph linear functions and show whole number intercepts;
- Match graphs of linear equations to tables of solutions;
- Describe the comparison of center (median, mean) of two different data sets.

Parma City School District

MATH PERFORMANCE LEVEL DESCRIPTORS – GEOMETRY

PROFICIENT	ACCELERATED	ADVANCED
A student performing at the **Proficient Level** demonstrates an appropriate command of Ohio's Learning Standards for Geometry. A student at this level has a consistent ability to use geometric transformations to prove congruence of triangles and theorems, to derive and use geometric relationships in similar triangles and trigonometric ratios in right triangles, to derive and use geometric measurements and relationships involving circles, and to understand and use concepts involving conditional probability and rules of probability.	A student performing at the **Accelerated Level** demonstrates a strong command of Ohio's Learning Standards for Geometry. A student at this level has a superior ability to use geometric transformations to prove congruence of triangles and theorems, to derive and use geometric relationships in similar triangles and trigonometric ratios in right triangles, to derive and use geometric measurements and relationships involving circles, and to understand and use concepts involving conditional probability and rules of probability.	A student performing at the **Advanced Level** demonstrates a distinguished command of Ohio's Learning Standards for Geometry. A student at this level has a sophisticated ability to use geometric transformations to prove congruence of triangles and theorems, to derive and use geometric relationships in similar triangles and trigonometric ratios in right triangles, to derive and use geometric measurements and relationships involving circles, and to understand and use concepts involving conditional probability and rules of probability.
A student at the **Proficient Level** can: • Recognize precise definitions of ray, angle, circle, perpendicular lines, parallel lines, and line segment based on the undefined notions of point, line, distance along a line and arc length; • Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure; • Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent; • Complete a proof of a theorem about lines and angles, triangles, or parallelograms requiring a routine proof;	A student at the **Accelerated Level** can: • Specify, using numeric values, a sequence of transformations that will carry a given figure onto another; • Analyze and correct a proof of a theorem about lines and angles, triangles, or parallelograms; • Use coordinates to prove/disprove that a given point lies on the circle centered on the origin and containing another given point; • Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of	A student at the **Advanced Level** can: • Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent; • Specify, using abstract values, a sequence , of transformations that will carry a given figure onto another; • Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions; • Construct a logical formal proof of a theorem about lines and angles, triangles, or parallelograms;

• Use coordinates to prove/disprove that a figure defined by four given points in the coordinate plane is a rectangle; • Use the slope criteria for parallel and perpendicular lines to solve geometric problems algebraically; • Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g. distance formula);Perform and/or explain a routine geometric construction procedure; • Complete a routine proof of a theorem involving proportionality of lengths within a triangle or among triangles; • Use congruence and similarity criteria for triangles to solve routine problems; • Explain the relationship between sine and cosine of complementary angles; • Use trigonometric ratios or the Pythagorean Theorem to solve routine real world problems involving right triangles; • Use congruence and similarity criteria for triangles to describe relationships in geometric figures; • Apply concepts of density based on volume in modeling situations; • Use geometric methods to solve routine design problems limited by constraints or restrictions; • Use the measures of geometric shapes and their properties to describe real-world objects; • Use the properties of similarity transformations to justify the AA criterion for two triangles to be similar;	angles and the proportionality of all corresponding pairs of sides; • Use triangle similarity to construct a proof of a theorem involving proportionality of lengths within a triangle or among triangles; • Use congruence and similarity criteria for triangles to prove relationships in geometric figures; • Use trigonometric ratios and the Pythagorean Theorem to solve routine real-world problems involving complex figures; • Use geometric methods to model design problems limited by constraints or restrictions; • Construct a proof of properties of angles for a quadrilateral inscribed in a circle; • Use the formula for the area of a sector of a circle to solve routine problems; • Use properties of circles to solve routine problems related to the equation of a circle, its center and radius length; • Give an informal argument for the formulas for the circumference and area of a circle; • Solve routine problems based on the volumes of compositions or parts of cylinders, pyramids, cones, or spheres; • Complete the square to find the center and radius of a circle given by an equation; • Construct inscribed and circumscribed circles of a triangle; • Describe events involving unions, intersections, or complements of other events using set notation;	• Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in non-routine real world problems; • Use a variety of geometric methods to model and solve non-routine design problems limited by many constraints or restrictions; • Describe the relationship among the measures of central angles, inscribed angles, and circumscribed angles; • Solve non-routine real world problems involving finding arc lengths and areas of sectors; • Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone; • Apply the understanding of scale factor to area and volume in solving a variety of non-routine problems; • Explain the concepts of conditional probability in everyday situations; • Apply the addition rule for probability for events that are not mutually exclusive and interpret the answer in terms of the model.

• Use transformations between two or more circles to show similarity; • Find the measures of inscribed angles and circumscribed angles given the measures of their intercepted arcs; • Write the equation of a circle given its center and radius; • Use volume formulas involving finding a measurement (e.g. height or radius) of cylinders, pyramids, cones, and spheres, given the volume and other measurements, to solve problems; • Determine the measure of an angle in degrees or radians given the arc length; • Describe events as unions, intersections, or complements of other events using the terminology "or," "and," "not"; • Determine the independence of two events in terms of the product of their probabilities; • Understand the conditional probability of A given B as P(A and B)/P(B); • Determine and interpret independence of two events using products of probabilities, conditional probabilities, and two-way frequency tables; • Recognize conditional probability in everyday language and everyday situations; • Apply the addition rule for probability for events that are not mutually exclusive.	• Use a two-way relative frequency table as a sample space to decide if events are independent by approximating conditional probabilities; • Explain the independence of events in everyday situations.	

Foundational Prerequisites

BASIC

A student performing at the **Basic Level** demonstrates partial command of Ohio's Learning Standards for Geometry. A student at this level has a general ability to use geometric transformations to prove congruence of triangles and theorems, to derive and use geometric relationships in similar triangles and trigonometric ratios in right triangles, to derive and use geometric measurements and relationships involving circles, and to understand and use concepts involving conditional probability and rules of probability.

A student whose performance falls within the **Basic Level** typically can:

- Carry out routine procedures;
- Solve simple problems using visual representations;
- Compute accurately some grade level numbers and operations;
- Recall and recognize some grade level mathematical concepts, terms and properties, and use more previous grade level mathematical concepts, terms and properties.

A student at the **Basic Level** can:

- Know precise definitions of ray, angle, circle, perpendicular lines, parallel lines, and line segment;
- Use geometric descriptions of rigid motions to transform figures;
- Given two triangles, determine if the two triangles are congruent based upon sides and angles;
- Complete a proof of a theorem about lines and angles, triangles, or parallelograms by identifying one or two statements or reasons missing from the proof;
- Use the slope criteria for parallel or perpendicular lines to solve geometric problems;
- Use coordinates to compute areas of triangles and rectangles with horizontal and/or vertical sides;
- Identify the step or steps needed to complete a given construction using a variety of tools and methods;
- Use scale factors to reduce and enlarge drawings on grids to produce dilations;
- Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar;
- Complete a straight-forward proof of a theorem involving proportionality of lengths within a triangle or among triangles by identifying one or two statements or reasons missing from the proof;
- Use congruence criteria for triangles to solve simple mathematical problems;
- Apply concepts of density based on area in modeling situations;
- Identify inscribed angles and circumscribed angles;
- Solve problems involving the volumes of cylinders, pyramids, cones, and spheres, given all necessary measurements;
- Identify the shapes of two-dimensional cross-sections of three-dimensional objects;
- Describe events as subsets of a sample space (the set of outcomes) using categories of the outcomes;
- Use two-way frequency tables of data when two categories are associated with each object being classified as a sample space to determine probabilities;
- Recognize independence of events in everyday language and everyday situations;
- Apply the addition rule for probability for events that are mutually exclusive;

LIMITED

A student performing at the **Limited Level** demonstrates a minimal command of Ohio's Learning Standards for Geometry. A student at this level has an emerging ability to use geometric transformations to prove congruence of triangles and theorems, to derive and use geometric relationships in similar triangles and trigonometric ratios in right triangles, to derive and use geometric measurements and relationships involving circles, and to understand and use concepts involving conditional probability and rules of probability.

A student whose performance falls within the **Limited Level** typically can:

- Carry out some routine procedures to solve straightforward one-step problems;
- Recognize solutions to some simple computation, straightforward problems;
- Compute accurately a few grade level numbers and operations;
- Recognize a few grade level mathematical concepts, terms and properties, and use previous grade level mathematical concepts, terms and properties.

A student at the **Limited Level** can:

- Recognize definitions of ray, angle, circle, perpendicular lines, parallel lines, and line segment;
- Given a geometric figure and a rotation, reflection, or translation, identify the transformed figure;
- Given four points on the coordinate plane, determine if the points create a rectangle with horizontal and vertical sides;
- Use slope criteria to determine if given lines are parallel or perpendicular;
- Use coordinates to compute perimeters of polygons with sides that are horizontal or vertical;
- Given a line segment length and a scale factor, determine the dilated line segment measure;
- Identify central angles and find their measures given the measure of their intercepted arcs;
- Recognize an equation of a circle;
- Use volume formulas to find volumes of cylinders, pyramids, cones, and spheres, given all needed measurements, to solve simple problems;
- Identify the sample space (the set of outcomes) using characteristics of the outcomes;
- Complete two-way frequency tables of data when two categories are associated with each object being classified;

MATERIALS & RESOURCE RECOMMENDATIONS

The content contained within this course guide outlines the district expectations for the sequencing of teaching and assessment activities throughout each school year for each grade/course. Specific material and/or resource recommendations will be communicated through the Dept. of Curriculum & Instruction at the beginning of each school year. Staff may also consult the Ohio Department of Education Model Curricula page by visiting the ODE web site and searching “Model Curriculum” for the subject specific documents.